Optimal ancilla-free phase-covariant telecloning of qudits via nonmaximally entangled states

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We study the $1 \to 2$ phase-covariant telecloning of a qudit without ancilla. We show that the fidelity of the two clones can reach that of the clones in the optimal ancilla-based $1 \to 2$ phase-covariant cloning and telecloning, i.e., the limitation of quantum mechanics. More interestingly, it is a nonmaximally entangled state rather than the maximally entangled state that can be used to realize such a telecloning task.

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It is impossible to exactly copy (that is, clone) an arbitrary quantum state because of the linearity of quantum mechanics [1, 2]. Nevertheless, the question of how well one can clone an unknown or partially unknown quantum state has been attracting much interest [3] since Bužek and Hillery [4] first introduced the concept of approximate quantum copying , because it is closely related to quantum computation, quantum communication, and quantum cryptography (see, e.g., [5–8]), and can also reveal some peculiar entanglement properties (see, e.g., [9–11]). If the input quantum state is chosen from a subset of linear independent states, exact copying can be realized probabilistically [12, 13]. For the input state $|\psi\rangle = \sum_{j=0}^{d-1} \alpha_j e^{i\theta_j}|j\rangle$ ($d \geq 2$ is the dimension) with α_j being real numbers satisfying the normalization condition $\sum_{j=0}^{d-1} \alpha_j^2 = 1$ and $\theta_j \in [0, 2\pi)$, three types of (approximate) quantum cloning have been intensively studied, i.e., universal quantum cloning with α_j and θ_j being completely unknown [14–16], real state cloning with $\theta_j = 0$ and α_j being unknown [17–19], and phase-covariant cloning with $\alpha_j = 1/\sqrt{d}$ and θ_j being unknown [17, 20]. In general, the more the information about the input state is known, the better the state can be cloned. As a consequence, the optimal fidelities of clones (the fidelity limit that quantum mechanics allows) in the real state cloning and phase-covariant cloning are higher than that in the universal quantum cloning. Recently, more attention was paid to phase-covariant cloning because of its use in connection with quantum cryptography [21].

Quantum cloning process can be regarded as distribution of quantum information from the initial system to a larger one. Thus quantum cloning combining with other quantum information processing tasks may have potential applications in quantum communication, distributed quantum computation, and so on [22, 23]. This leads to the advent of the concept of telecloning [24], which is the combination of quantum cloning and quantum teleportation [25]. Telecloning functions as transmitting multiple copies of an unknown (or partially unknown) quantum state to distant sites, i.e., realizing one-to-many nonlocal cloning, via previously shared multipartite entangled states. The entanglement channel for telecloning can be directly constructed by the corresponding cloning transformation [26].

In the aforementioned quantum cloning and telecloning, the ancillas (extra quantum systems besides the ones used to carry the cloned states) plays an important role. Recently, quantum cloning without ancillas, i.e., the so-called ancilla-free (or economical) cloning [27–30], has attracted much interest, because it may be easier than the one with ancillas for experimental implementation [31]. Durt *et al.* showed that an ancilla-free version of the $1 \to 2$ universal cloning with the optimal fidelity (the fidelity limit that quantum mechanics allows) cannot be realized in any dimension, and ancilla-free versions of both the $1 \to 2$ Fourier-covariant [32] and phase-covariant cloning with the optimal fidelity can be implemented only for qubits. They also presented an ancilla-free phase-covariant cloning machine for qudits, with the fidelity being lower than that of the optimal phase-covariant cloning machine involving an ancilla. Because of the relationship between cloning and the corresponding telecloning [26, 33, 34], their conclusions also imply that the ancilla-free $1 \to 2$ phase-covariant telecloning with the optimal fidelity for qudits and universal telecloning with the optimal fidelity in any dimension cannot be realized via the maximally entangled states constructed by the corresponding cloning transformations.

In this letter, we present a scheme for ancilla-free $1 \to 2$ phase-covariant telecloning of a qudit. We show that the fidelity can reach that of the $1 \to 2$ phase-covariant cloning machine of Ref. [20] and telecloning machine of [33]. That is, the fidelity of the clones in our ancilla-free telecloning scheme can hit to the optimal fidelity that quantum mechanics allowed. More interestingly, the suitable quantum channel for realizing the above telecloning task is a nonmaximally entangled state rather than the maximally entangled state.

First, we briefly review Durt's ancilla-free $1 \to 2$ phase-covariant (symmetric) cloning machine for a d-dimensional system. For the input state

$$|\psi^{in}\rangle_1 = \frac{1}{\sqrt{d}} \sum_{j=0}^{d-1} e^{i\theta_j}|j_1\rangle, \tag{1}$$

the cloning machine (transformation) functions as [30]

$$|j_1 0_2\rangle \to |\phi_j^0\rangle_{12} \tag{2}$$

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where

$$|\phi_k^k\rangle_{12} = |k_1 k_2\rangle$$
$$|\phi_j^k\rangle_{12} = \frac{1}{\sqrt{2}}(|j_1 k_2\rangle + |k_1 j_2\rangle),\ j \neq k. \quad (3)$$

Here, we have assumed that the second quantum system (carrier) is initially in the state $|0_2\rangle$. The output state reads

$$|\psi^{out}\rangle_{12} = \frac{1}{\sqrt{d}} \sum_{j=0}^{d-1} e^{i\theta_j} |\phi_j^0\rangle_{12}. \quad (4)$$

The fidelity of each clone (copy) is

$$\begin{aligned} F_{econ}(d) &= \langle\psi^{in}|_{1(2)} \mathrm{Tr}_{2(1)} \left(|\psi^{out}\rangle_{12}\langle\psi^{out}|\right) |\psi^{in}\rangle_{1(2)} \\ &= \frac{1}{2d^2}\left[(d-1)^2 + (1+2\sqrt{2})(d-1) + 2\right]. \end{aligned} \quad (5)$$

However, the optimal fidelity of $1 \to 2$ phase-covariant cloning (with an ancilla) is [20]

$$F_{opt}(d) = \frac{1}{4d}\left(d + 2 + \sqrt{d^2 + 4d - 4}\right). \quad (6)$$

It can be verified that for $d = 2$, $F_{econ}(2) = F_{opt}(2)$, while for $d > 2$, $F_{econ}(d) < F_{opt}(d)$. Thus this type of ancilla-free phase-covariant cloning is "suboptimal".

We now describe our telecloning protocol. The task is: Alice wants to transmit one copy of the state $|\psi^{in}\rangle_{A_1}$ of particle A_1 to distant Bob and Charlie, respectively. Assume that the quantum channel among them is a three-particle entangled state as follows:

$$|\Psi\rangle_{A_2BC} = \sum_{j=0}^{d-1} x_j |j_{A_2}\rangle |\phi_j^0\rangle_{BC}, \quad (7)$$

where x_j are probability amplitudes satisfying normalization condition $\sum_{j=0}^{d-1} x_j^2 = 1$. For simplicity, we have assumed that x_j are real numbers. Here, particle A_2 is on Alice's hand, and particles B and C are held by Bob and Charlie, respectively. The von Neumann entropy of $\rho_{A_2} = \mathrm{tr}_{BC}(|\Psi\rangle_{A_2BC}\langle\Psi|)$ is

$$S(\rho_{A_2}) = -\sum_{j=0}^{d-1} x_j^2 \log_2 x_j^2. \quad (8)$$

The state of the total system is

$$\begin{aligned} |\Psi\rangle_{total} &= |\psi^{in}\rangle_{A_1} \otimes |\Psi\rangle_{A_2BC} \\ &= \frac{1}{d}\sum_{l=0}^{d-1}\sum_{k=0}^{d-1} |\Phi\rangle_{A_1A_2}^{lk} \\ &\quad \times \sum_{j=0}^{d-1} e^{-2\pi ijk/d} x_{j\oplus l} e^{i\theta_j} |\phi_{j\oplus l}^0\rangle_{BC}, \end{aligned} \quad (9)$$

where $j \oplus l$ denotes $j + l$ modulo d and $|\Phi\rangle_{A_1A_2}^{lk}$ are generalized Bell-basis states given by

$$|\Phi\rangle_{A_1A_2}^{lk} = \frac{1}{\sqrt{d}}\sum_{j=0}^{d-1} \exp\left(\frac{2\pi ijk}{d}\right)|j\rangle|j\oplus l\rangle. \quad (10)$$

In order to realize the telecloning task, Alice performs a complete projective measurement jointly on particles A_1 and A_2 in the generalized Bell-basis $\{|\Phi\rangle_{A_1A_2}^{lk},\ l,k = 0,1,2\cdots,d-1\}$, and informs Bobs of the outcome.

If Alice gets the outcomes $|\Phi\rangle_{A_1A_2}^{0k}$, the state of particles B and C collapses into

$$|\psi\rangle_{BC} = \sum_{j=0}^{d-1} e^{-2\pi ijk/d} x_j e^{i\theta_j} |\phi_j^0\rangle_{BC}. \quad (11)$$

After receiving the measurement outcome, Bob and Charlie perform, respectively, their particles the following local operation:

$$U_{B(C)}^{0k} = \sum_{j=0}^{d-1} \exp\left(\frac{2\pi ijk}{d}\right)|j\rangle_{B(C)}\langle j|. \quad (12)$$

Then the state of Eq. (11) evolves into

$$|\psi^{out'}\rangle_{BC} = \sum_{j=0}^{d-1} x_j e^{i\theta_j}|\phi_j^0\rangle_{BC}. \quad (13)$$

The fidelity of clones that Bob and Charlie obtained is

$$F_{econ}^t(d) = \frac{1}{d}\left(1 + \sqrt{2}x_0\sum_{j=1}^{d-1} x_j + \sum_{j=1}^{d-2}\sum_{k=j+1}^{d-1} x_j x_k\right). \quad (14)$$

If Alice gets the outcome $|\Phi\rangle_{A_1A_2}^{lk}$ ($l \neq 0$), the state of particles $\{1,2,\cdots,n\}$ collapses into

$$|\tilde{\psi}\rangle_{BC} = \sum_{j=0}^{d-1} e^{-i2\pi jk/d} x_{j\oplus l} e^{i\theta_j}|\phi_{j\oplus l}^0\rangle_{BC}. \quad (15)$$

After receiving the measurement outcome, Bobs perform, respectively, their particles the following local operation:

$$U_{B(C)}^{lk} = \sum_{j=0}^{d-1} \exp\left(i\frac{2\pi jk}{d}\right)|j\rangle_{B(C)}\langle j\oplus l|. \quad (16)$$

The state of Eq. (15) evolves into

$$|\psi\rangle_{BC}^{out''} = \sum_{j=0}^{d-1} x_{j\oplus l} e^{i\theta_j}|\phi_j^{d-l}\rangle_{BC}, \quad (17)$$

where a nonsense global phase factor $\exp[i2\pi(d-l)k/d]$ is discarded. It can be easily verified that $\{x_{j\oplus l}|\phi_j^{d-l}\rangle_{BC},\ j = 0,1,\cdots,d-1\}$ have the same contributions with $\{x_j|\phi_j^0\rangle_{BC}\}$ to the cloning fidelity. Thus the fidelity of each clone is also equal to $F_{econ}^t(d)$ in this case. Unlike Ref. [26], our protocol does not involve ancilla and thus it is ancilla-free.

For the case $x_j = 1/\sqrt{d}$, $S(\rho_{A_2}) = \log_2 d$ and the quantum channel is a maximally entangled state in terms of the subsystem of Alice (particle A_2) and the subsystem of Bob and Charlie (particles B and C). Then $F_{econ}^t(d) = F_{econ}(d)$ less than $F_{opt}(d)$ for $d > 2$. In the following, we shall show

that the fidelity $F^t_{econ}(d)$ can be equal to $F_{opt}(d)$ for any d with another choice of $\{x_j\}$.

We set

$$x_0 = X(d) = \sqrt{\frac{4(d-1)}{D(D+d-2)}},$$
$$x_j = Y(d) = \sqrt{\frac{d^2+(d-2)D}{D(D+d-2)(d-1)}},\ j \neq 0, \quad (18)$$

where $D = \sqrt{d^2+4d-4}$. Then it can be verified that $F^t_{econ}(d) = F_{opt}(d)$ for any d. In fact, the output state $(|\psi^{out'}\rangle_{BC}\langle\psi^{out'}|)$ of our telecloner is then equivalent to that (ρ^{out}_{opt}) of the optimal phase-covariant cloner after tracing out the ancilla [20, 21]. Particularly, $\rho^{out}_{opt} = |\psi^{out'}\rangle_{BC}\langle\psi^{out'}| + \tilde{\rho}$ with $\langle\psi^{in}|_{B(A)}\mathrm{tr}_{A(B)}(\tilde{\rho})|\psi^{in}\rangle_{B(A)} = 0$. In this case, the entanglement channel of Eq. (7) reduces to

$$|\Psi'\rangle_{A_2BC} = X(d)|0_{A_2}\rangle|\phi^0_0\rangle_{BC} + Y(d)\sum_{j=1}^{d-1}|j_{A_2}\rangle|\phi^0_j\rangle_{BC}. \quad (19)$$

When $d = 2$, $S(\rho_{A_2}) = 1$ and $|\Psi'\rangle_{A_2BC}$ is a maximally entangled state. For $d > 2$, however, the amount of entanglement with von Neumann measure between particle A_2 and particles (B,C) is $E(|\Psi'\rangle_{A_2(BC)}) = -X^2\log_2 X^2 - (d-1)Y^2\log_2 Y^2 < \log_2 d$, which implies that the subsystem of Alice (sender) and the subsystem of Bob and Charlie (receivers) in the state of Eq. (19) are only partially entangled. Thus we can safely conclude that the ancilla-free $1 \to 2$ phase-covariant telecloning with the optimal fidelity for qudits can be realized via suitable nonmaximally entangled states acting as the quantum channel.

In order to reveal clearly the relationship between the fidelity of clones and the amount of entanglement of the quantum channel, we show how $F^t_{econ}(d)$ varies with the variation of von Neumann entropy $S(\rho_{A_2})$ in Fig. 1. For simplicity, we have assumed that $x_1 = x_2 = \cdots = x_{d-1}$. It can be seen that for $d = 2$, the increase (decrease) in $S(\rho_{A_2})$ always leads to increase (decrease) in $F^t_{econ}(2)$. For $d > 2$, however, a counterintuitive phenomenon appears: when $1/\sqrt{d} \leq x_0 \leq X(d)$, $F^t_{econ}(d)$ increases (decreases) with the decrease (increase) in $S(\rho_{A_2})$.

In conclusion, we have studied the ancilla-free $1 \to 2$ phase-covariant telecloning for qudits. We have shown that the fidelity can reach that of the $1 \to 2$ phase-covariant cloning machine of Ref. [20]. In other words, the fidelity of the clones in our ancilla-free teleclonig scheme can hit to the optimal fidelity (the fidelity limit that quantum mechanics allows for phase-covariant cloning). We have also shown that the increase (decrease) in amount of entanglement of the quantum channel may lead to the decrease (increase) in the fidelity of clones in the ancilla-free phase-covariant telecloning for qudits. This effect leads to another interesting phenomenon: the suitable quantum channels for realizing the optimal ancilla-free $1 \to 2$ phase-covariant telecloning of qudits are special configurations of nonmaximally entangled states rather than the maximally entangled states. Note that nonmaximally entangled states can be better than the maximally entangled states for several other quantum tasks has also been reported [35].

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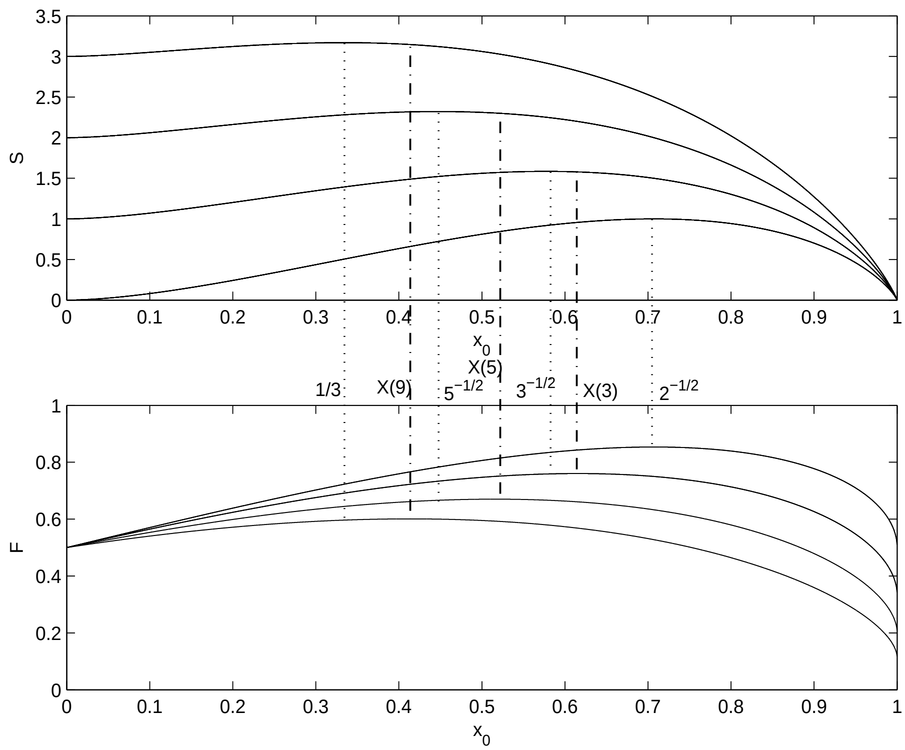

FIG. 1: The von Neumann entropy $S(\rho_{A_2})$ (upper graph) and the fidelity $F^t_{econ}(d)$ (lower graph) versus the probability amplitude x_0, where $x_1 = x_2 = \cdots = x_{d-1}$. From bottom (top) to top (bottom) in the upper (lower) graph, the curves correspond to $d = 2, 3, 5$, and 9, respectively. The vertical dotted lines ending in the corresponding curves represent that S reaches the maximum when $x_0 = 1/\sqrt{d}$, and the dashdotted lines denote that F hits to the maximum when $x_0 = X(d)$.